Second-order Taylor polynomial for a production function

Given the following production function: $f(L, K) = 3L^{\frac{2}{3}}K^{\frac{1}{3}}$

1. Develop the Taylor polynomial around the point $P = (8, 27)$ up to the second order.
2. Calculate an approximate value of the production if the labor is increased by 2% and the capital by 1%.

Solution

1. Formula, for a given point (a, b)

$$f(x, y) \approx f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) + \frac{1}{2} f_{xx}(a, b)(x-a)^2 + f_{xy}(a, b)(x-a)(y-b) + \frac{1}{2} f_{yy}(a, b)(y-b)^2$$

   To develop the Taylor polynomial, one must calculate the first and second order partial derivatives and replace them at the given point. With these obtained values, the following polynomial is obtained:

$$P_2(L, K) = 36 + 3(L-8) + \frac{4}{9}(K-27) + \frac{1}{2}\left(\frac{1}{8}(L-8)^2\right) + \frac{2}{27}(L-8)(K-27) - \frac{8}{729}(K-27)^2$$

2. Then, an approximate value is calculated at the point $P_0 = (8 \cdot 1.02, 27 \cdot 1.01)$

$$P_2(8.16, 27.27) = 36.5996$$